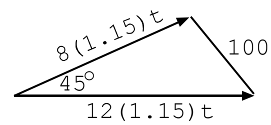
2013 – 2014 Study List Solutions

08G-36. Two boats leave each other, one traveling northeast at 8 knots and the other traveling east at 12 knots. How long does it take them to be 100 mi apart if a knot is 1.15 mph? -------------------------------- 36= ________________ hr

$100^2 = (9.2t)^2 + (13.8t)^2 - 2(9.2t)(13.8t)\cos 45°$

Solver: t = **10.2**



09H-36. A 36-in piece of string is used to form a triangle with sides in the ratio 4:5:8. What is the triangle area? ---------------------------- 36= ________________ in^2

4 + 5 + 8 = 17

Side 1 = (4/17)(36) = 8.470… *Side 2* = (5/17)(36) = 10.588…

Side 3 = (8/17)(36) = 16.941… Semi-perimeter = 36/2 = 18

Area = $\sqrt{(18)(18-16.941...)(18-10.588...)(18-8.470...)}$ = **36.7**

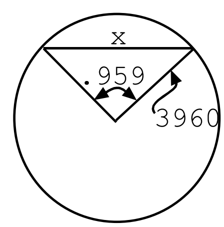
10A-37. Traveling on the earth's surface, it is 3800 mi from Dallas to Hawaii. What is the percent difference between this arc length and the "burrow-through the earth" straight line distance? -----------37= ________________ %

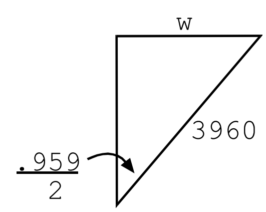
$s = \theta r$ $\theta = s / r = 3800 / 3960 = .95959596$ rad

$\sin(\theta/2) = w / 3960$ $w = 1827.9359$

$x = 2w = 3655.8718$

3800, 3655.8718, %chg = **- 3.79**





Note: percent *decrease* would be a positive answer.

10A-38. One point (a, b) on the line y = 5x + 3 is equidistant from the points (4, 8) and (-3, -5). What is a?---------------------------------38= ____________________

$$\sqrt{(x-4)^2+(y-8)^2} = \sqrt{(x+3)^2+(y+5)^2}$$

$-8x + 16 - 16y + 64 = 6x + 9 + 10y + 25$

$-26y = 14x - 46$

$y = (14x - 46) / -26$ and $y = 5x + 3$

x **- 0.222**

10B-38. An artilleryman fires a shell at an angle relative to the horizontal of 27 degrees. The shell reaches a maximum elevation of 68 ft but falls 40 ft short of the target. What new angle less than 45 degrees should the artilleryman use to hit the target on the second shot?--- 38= ________________deg

$$68 = \frac{v^2 \sin^2 27^\circ}{2(32.17)} \qquad v = 145.696\ldots \text{ ft/s}$$

$$d_{h_{max}} = \frac{(145.696...)^2 \sin 54^\circ}{32.17} = 533.83$$

$$533.83 + 40 = \frac{(145.696...)^2 \sin\theta}{32.17} \qquad \text{Solve } \theta = 60.416$$

½ θ = **30.2**

10E-37. A mother sees her small child running away from her and begins to give chase to catch her. The child was 30 ft away and was moving at 4 ft/s. If the mom can run a mile in 8 min 25 s, how long will it take for her to catch her child? ---------------------------- 37= __________________s

$$\left(\frac{60}{8\,\frac{25}{60}}\right)\left(\frac{22}{15}\right)t = 30 + 4t \qquad t = \mathbf{4.65}$$

10F-36. How far is it from Austin TX to Istanbul if a plane averaging 560 mph leaves Austin at 3:30 AM local time and arrives in Istanbul, 8 time zones later, at 11 PM local time the same day? -- 36=________________ mi

Keeping the time as "Austin time": 3:30 AM to 3 PM = 11 ½ hrs

D = r × t = (560mph) (11 ½ hr) = **6440**

10G-37. Andrea walked 0.88 mi one day. The next day she walked 88% of 0.88 miles. The third day, she walked 88% of the preceding day's distance. How far total will she eventually have walked?---- 37= ________________ mi

Infinite geometric series sum:

s = .88 ÷ (1 - .88) = **7.33**

11B-37. A naval electric gun can be designed to fire a projectile at Mach 7.5, reaching a maximum vertical height of 500,000 ft. What is the maximum horizontal range of the projectile? The Mach number is the projectile velocity divided by the speed of sound, 1116 ft/s.--------- 37= _______________ mi

$v = 7.5(1116) = 8370$

$$d_{v_{max}} = \frac{v^2 \sin^2\theta}{2g} \qquad 500{,}000 = \frac{8370^2 \sin^2\theta}{2(32.17)} \qquad \theta = 42.659\ldots$$

$$d_{h_{max}} = \frac{v^2 \sin 2\theta}{g} \qquad d_{h_{max}} = \frac{8370^2 \sin 2(42.659)}{32.17} \div 5280 = \mathbf{411}$$

11D-36. A parachute is designed to automatically deploy when the freefall velocity reaches 65 mph. At what elevation should a plane fly if the parachute opens at 8,000 ft? --- 36= ________________ ft

$V^2 = V_0^2 + 2a(y - y_0)$ $\qquad$ 65 mph × 88/60 = 95.33…ft/s

$(95.3)^2 = 0 + 2(32.17)y$

$y = 141.2565\ldots$ $\qquad$ $8000 + 141.2565 =$ **8140**

11E-36. A ball is dropped vertically from a height of 1 yard. It recovers 80% of its height on the first bounce. What is the total distance traveled by the ball from the time it was released until it came to rest on the floor? -- 36= ________________ ft

$s = 3 / (1 - .8) = 15$ $\qquad$ $2(15) - 3 =$ **27.0**

11F-36. In a 3-person relay race, each makes 2 laps around a ¼ mi track. The first person did his laps at a "rate" of 6 min 23.6 s per mi. The second person pulled a muscle but still finished his laps in 11 min 34 s. The last person ran his 2 laps at a 5 min 52.3 s per mi pace. What was their total time ?--- 36= ___________min(SD)

Rate P_1 = [6(60) + 23.6)]/mi
Rate P_2 = [11(60)+34]/(1/2 mi)
Rate P_3 = [5(60) + 52.3]/mi $\qquad$ But each person only did ½ mi, so

total time = [6(60) + 23.6)]/2 + [11(60)+34] + [5(60) + 52.3]/2 = 1061.95 sec
1061.95 sec / (60 sec/min) {4SD} = **17.70**

11G-36. A 10-in long, round candle is tapered, 0.75 inches in diameter at the bottom and 0.45 inches in diameter at the top. At what rate is the wax consumed if burning 80% of the candle takes 11 hr? ------------ 36= ____________ in^3/hr

$V = (1/3)\pi\,[r_1^2 + r_2^2 + r_1 r_2]\,h$

$$V = \frac{1}{3}\pi\left[\left(\frac{.75}{2}\right)^2 + \left(\frac{.45}{2}\right)^2 + \left(\frac{.75\times.45}{2\times2}\right)\right]10$$

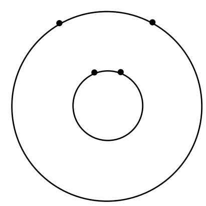
V = 2.886… .8V / 11 = .**210**

11I-37. Mercury orbits the sun every 88.0 dy, Jupiter every 11.87 yr and Uranus every 84.07 yr. How many trips around the sun does Mercury make between the times of closest approach of Jupiter and Uranus? All orbits are in the same direction. ------------------------------- 37= _____________(SD)

(1/11.87)t = (1/84.7)t + 1

t = 13.8214… yr

t / (88/365.256) = **57.4**



12A-38. Rodney made punch by adding 20 tablespoons of concentrate to a gallon bottle and then topped off the bottle with water. Later, he learned that it should have been 20 teaspoons of concentrate. How much punch should be poured out of the bottle so that the concentration is correct when the bottle is topped off with water? -- 38= _____________ cups

1 tablespoon = 3 teaspoons
2 tablespoons = 1 ounce
128 oz = 1 gal

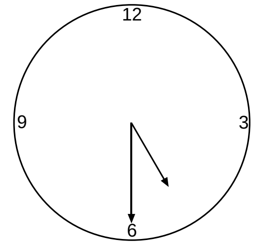
20 tablespoons = 10 ounces

10/128 should have been 10/[3(128)]

Pour out 2/3 of bottle.

2/3 gallon = 2/3(16 cups) = **10.7**

12D-36. How long after 5:30 do the minute and hour hands of a clock first align? -- 36= _________________ min



$11/12\ T = 30 + 27.5 \quad T = \mathbf{62.7}$

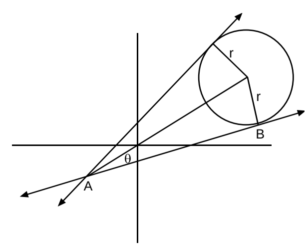
Alternate Solution:

$$\frac{11\left(30^\circ\right)+{}^{30}\!/_{60}\left(30^\circ\right)}{5.5^\circ\ /\ \text{min}}$$

12D-37. Tracy can sew a Christmas ornament in 4 min, and Carol can sew one In 2.7 min. Starting at 8 AM, Tracy works alone for time t and is then joined by Carol. What is t if they finished 120 ornaments at noon? ------------37= ___________________ hr

$(1/4)(4 \times 60) + (1/2.7)(x \times 60) = 120 \qquad x = 2.700 \qquad 4 - x =$ **1.30**

12E-38. What is the radius of a circle tangent to the lines y = 3x+7 and y = 0.5x-3 and containing the point (8,1)? ----------------------------------38= ______________________



Find angle between lines, divide by 2.
Find point of intersect of lines (A)
Find distance between (A) and (B)
Find r.

$\tan\theta = (m_1 - m_2) / (1 + m_1 m_2) \qquad m_1 = 3, \quad m_2 = .5$
$\theta = 45°, \quad \theta/2 = 22.5°$

$3x + 7 = .5x - 3 \qquad x = -4, \quad y = -5$

$$d = \sqrt{[8-(-4)]^2 + [1-(-5)]^2} = 13.4164\ldots$$

$\tan 22.5° = r / d \qquad r = \mathbf{5.56}$

12F-38. Steve mixed 5 cups of milk with 6 cups flour to make crepe batter. After using 65% of the batter, he decided to make some pancakes with the rest. Since pancake batter is thicker, he needed to add more flour. If the pancake recipe calls for 1 part milk to 2 parts flour, how much flour should Steve add? 38= _______ cups

$$\frac{6(.35)+x}{11(.35)+x} = \frac{2}{3} \qquad x = \mathbf{1.40}$$

12H-36. Ganymede, a moon of Jupiter, is the largest moon in the solar system. Its surface area is 0.171 the surface area of earth. What is the percent error in reporting the volume of Ganymede as 0.07 times the volume of earth?-------- 36= __________ %

$(.171)^{3/2} = .07071217$ .07071217 , .07, % chg = **-1.01**

13A-37. Brad left San Saba on State Highway 190 driving to Iraan, 204 mi away, at 53 mph. Brandon left Iraan 30 min after Brad left, driving to San Saba on the same highway. If they met in Eldorado which is 79 mi from Iraan, what was Brandon's velocity ? --37= __________ mph

$53t = 204 - 79 \quad \rightarrow \quad t = 2.3585$

$V(t - .5) = 79$

V = **42.5**

13B-38. A golfer drives a ball 120 ft using an 8 iron, not counting the ball roll after landing. What is the impact velocity of the 8 iron on the ball if an 8 iron launches a ball at an angle of 37 degrees relative to the ground? -------------38= __________ mph

$d_{h_{max}} = \dfrac{v^2 \sin 2\theta}{g}$ $\quad 120 = \dfrac{v^2 \sin 74^\circ}{32.174}$ $\quad$ v = 63.3756… ft/s x (60/88) = **43.2**

13C-38. A spring elongates 1 in for every 5 lbs of load. Four gallons of coconut oil (density equals 0.92 g/cm^3) are hung on the spring which is attached to a frame. However, the container has a leak, losing 10 tablespoons of coconut oil every minute. How long will it take for the container to rise 1.875 in? ----- 38= ___________ hr

.92 g x (1 kg/ 1000g) → .002028253 pounds

cm^3 x (1 ml/1cm^3) x (1 L/1000mL) → .0002641 gal 7.6777… lb/gal

F = (constant)(distance) F = kx 5 = k(1) k = 5

F = (5)(1.875 in) = 9.375 lb

9.375 lb / 7.6777…lb/gal = 1.221057… gal

(1.221057…gal)(128 oz/gal)(2 Tbsp/oz)(1 min/10 Tbsp)(1 hr/60 min) = **.521**

13D-36. The velocity of a solid sphere sinking in water is proportional to the square of its diameter. A 1 in diameter sphere sinks at 0.5 m/s. If this sphere were divided into N smaller spheres that each sank at 20 mm/s, what is N? 36=________ integer

$500 / 1^2 = 20 / d^2 \quad d = .2$

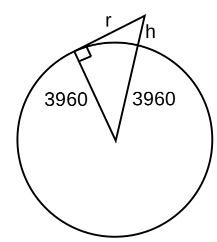
$V_1 / V2 = [4/3\pi(1)^3 \; / \; 4/3\pi(.2)^3] =$ **125**

13G-37. The probability of a person being struck by lightning during their lifetime is 1/280,000. What is the probability of being struck by lightning 1000 times in one's lifetime?---37= ______________

$(1 / 280{,}000)^{1000} \quad \rightarrow \quad 1000 \log (1 / 280{,}000) = -5447.15803\ldots$

$-5447.15803\ldots + 5448 = .84196\ldots \quad \rightarrow 10^{.84196\ldots} = 6.95 \quad \rightarrow$ $\mathbf{6.95 \times 10^{-5448}}$

13G-38. How high a tower must Ranger Stan climb to see a land area of $100\ mi^2$? --- 38= ___________ ft

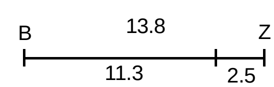


$\pi r^2 = 100$
$r = 5.64189\ldots$

$3960^2 + r^2 = (3960 + h)^2$
$h = .004019$ mi

$5280h =$ **21.2**

13H-37. Barry leaves Brownwood, running at 8.5 min/mi pace to Zephyr which is 13.8 mi away. After time t, Zane leaves Brownwood by bicycle, looking for Barry. What is t if Zane bikes at 18 mph? They met up 2.5 mi before Zephyr. --------- --37=___________min



$T_1 = 11.3 \text{ mi} / 18 \text{ mph} = .6277\ldots \text{ hr}$

$T_2 = 11.3 \text{ mi} / (60/8.5)\text{mph} = 1.600833\ldots \text{ hr}$

$(T_2 - T_1)\, 60 =$ **58.4**

13I-36. Once 70% of the entire state of Texas received an average of <u>1.3</u>-in rainfall. What total volume of rain was deposited if the area of the state is <u>268,581</u> mi^2? --- 36= _______ mi^3(SD)

$$V = \frac{\left[(268{,}581\ mi^2)(.7)(13")\right]}{\left[(12"/ft)(5280\,ft/mi)\right]} = \mathbf{3.9}\ \text{(2SD)} \qquad \{1.3 \text{ is 2 SD}, \ \ 268{,}581 \text{ is 6 SD}\}$$

13I-37. A certain insect population grows exponentially. The first population estimate was 8500 and ten years later it was 136,000. Over what time period did the insect population triple? ---37= ____________ yr

$A = A_0\, e^{kt}$

$136{,}000 = 8500\, e^{k(10)}$

$k = [\ln(1360 / 85)] / 10$

$e^{kt} = 3$ when $t =$ **3.96**

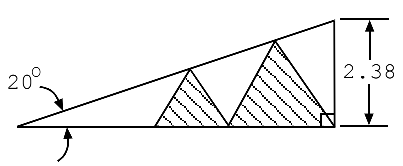
13I-38. Two runners start together running laps on a 400-meter track. The faster runner ran at 4.5 m/s, and she “lapped” the slower runner in 4 min 35 s. How far did the slower runner run? --- 38= ___________ m

4.5 (275 sec) = x + 400

x = **838**

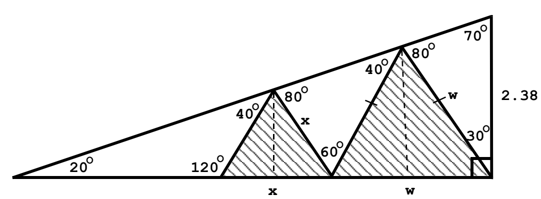
08A-60.

RIGHT AND EQUILATERAL TRIANGLES



HATCHED AREA = ?

08A-60 = ______________________



90° - 20° = 70°

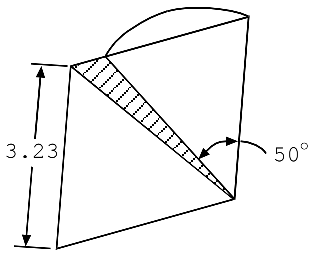
sin 80° / 2.38 = sin 70° / w
w = 2.27…

sin 80° / w = sin 40° / x
x = 1.48…

$$\text{hatched area} = \frac{x^2\sqrt{3}}{4} + \frac{w^2\sqrt{3}}{4} = \mathbf{3.18}$$

08D-60.

RHOMBUS AND SECTOR



HATCHED AREA = ?

08D-60 = ______________________

Notice △ on right is isosceles, so top right angle is $(180° - 50°)/2 = 65°$.

Then bottom left angle is also 65°.

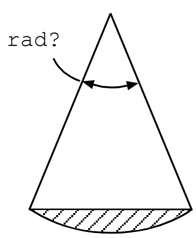
Shaded Area

$= ½ (3.23)^2 \sin 65° - ½ (3.23)^2 \sin 50°$

$=$ **.732**

08G-60.

ISOSCELES TRIANGLE AND SEGMENT



AREA(TRIANGLE) = 2190
AREA(SEGMENT) = 209

08G-60 = ______________________

$A_{triangle} = ½ ab \sin C$

$2190 = ½ r^2 \sin \theta$

$\sin \theta = 4380 / r^2$

$A_{segment} = (r^2/2)(\theta - \sin \theta) = 209$

$418 = r^2\theta - r^2 \sin \theta$

$418 = r^2\theta - r^2 (4380 / r^2)$

$418 = r^2\theta - 4380$

$\theta = 4798 / r^2$

$\sin (4798/r^2) = (4380/r^2)$

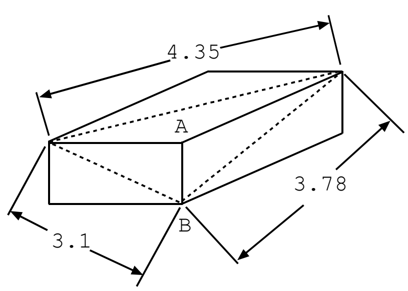
when $0 < r < (\pi/2)$

Solver: $r = 80.918\ldots$

$\theta = 4798/r^2 =$ **.733**

09E-50.

RECTANGULAR SOLID



AB = ?

09E-50 = ______________________

$x^2 + y^2 = 3.1^2$ $\qquad y^2 = 3.1^2 - x^2$

$x^2 + z^2 = 3.78^2$ $\qquad z^2 = 3.78^2 - x^2$

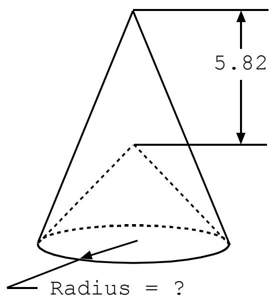
$y^2 + z^2 = 4.35^2$

$(3.1^2 - x^2) + (3.78^2 - x^2) = 4.35^2$

Solver: $x =$ **1.58**

09H-50.

CONE WITH CONICAL CAVITY



Volume = 200

09H-50 = ________________________

$$200 = (1/3)\,\pi\, r^2\, h_1 - (1/3)\,\pi\, r^2\, h_2$$

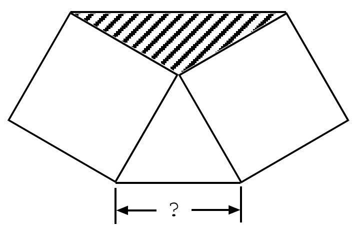
$$200 = (1/3)\,\pi\, r^2\, (h - (h - 5.82))$$

$$200 = (1/3)\,\pi\, r^2\, (5.82)$$

r = **5.73**

10B-60.

EQUILATERAL AND ISOSCELES TRIANGLE AND SQUARES



Hatched Area = 530

10B-60 = ________________________

$$360° - (60 + 90 + 90) = 120°$$

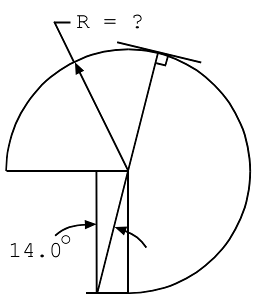
$$A = ½\, ab \sin C$$

$$530 = ½\, x^2 \sin 120°$$

x = **35.0**

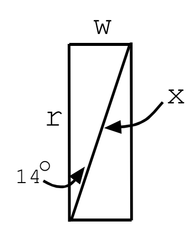
10D-60.

THREE-QUARTER CIRCLE AND RECTANGLE



Total Area = 570

10D-60 = ________________________



$$w = r \tan 14°$$

height = r

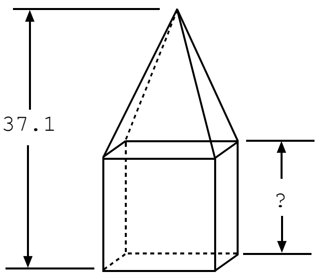
$$\text{rect} = wr = r^2 \tan 14°$$

$$570 = ¾\, \pi r^2 + r^2 \tan 14°$$

r = **14.8**

10E-50.

CUBE AND SQUARE PYRAMID



Pyramid Volume $= \frac{1}{2}\left[\text{Cube Volume}\right]$

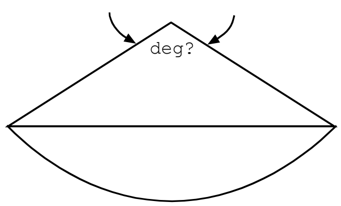
10E-50 = ________________________

$1/3\ x^2(37.1 - x) = ½(x)^3$

$x = \mathbf{14.8}$

11B-60.

SECTOR AND SEGMENT



Segment Area = Triangle Area

11B-60 = ________________________

$A_1 = (r^2/2)(\theta - \sin\theta) \qquad A_2 = ½\ r^2 \sin\theta$

$(r^2/2)(\theta - \sin\theta) = (r^2/2)\sin\theta$

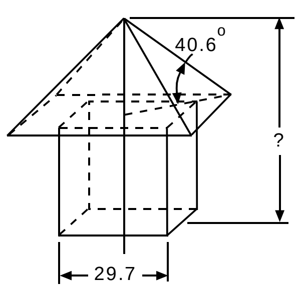
$\theta - \sin\theta = \sin\theta$

$\theta = 2\sin\theta$

$\theta = 1.89549\ldots \text{ rad} = \mathbf{109°}$

12A-50.

CUBE AND SQUARE PYRAMID



Total Volume = 38,700

12A-50 = ________________________

$V = V_1 + V_2 \qquad V = x^3 + 1/3\ Bh$

Let w = bottom edge of pyramid, h = height

$\tan 40.6° = h / \frac{w\sqrt{2}}{2} = h / \left(\frac{w}{\sqrt{2}}\right)$

$h = (w)\left(\frac{\tan 40.6°}{\sqrt{2}}\right)$

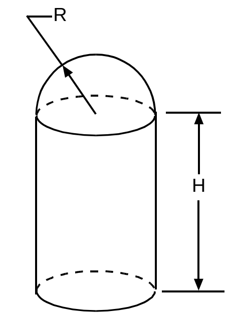
$38{,}700 = 29.7^3 + (1/3)w^2\ (w)\left(\frac{\tan 40.6°}{\sqrt{2}}\right)$

$w = 39.5542\ldots \qquad h = 23.9724\ldots.$

$h + 29.7 = \mathbf{53.7}$

12D-50

CYLINDER AND HEMISPHERE



Total volume = $10.5R^3$

H/R = ?

12D-50 = ________________________

$$V = \pi r^2 h + (2/3)\pi r^3$$

$$V = \pi R^2 H + (2/3)\pi R^3 = 10.5 R^3$$

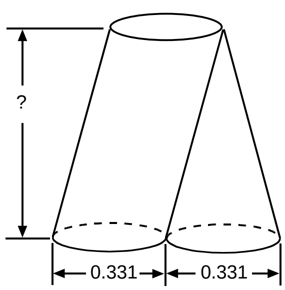
Let R = 1.

$$\pi H + (2/3)\pi = 10.5$$

$$H = \frac{10.5 - \frac{2}{3}\pi}{\pi} = \mathbf{2.68}$$

12E-49.

SLANT CIRCULAR CYLINDER AND CONE



Combined Total Volume = 0.0634

12E-49 = ________________________

$$V = \pi r^2 h + (1/3)\pi r^2 h$$

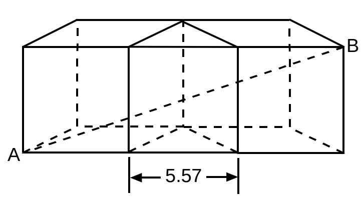
$$V = (4/3)\pi r^2 h$$

$$.0634 = (4/3)\pi (.331/2)^2 h$$

$$h = \mathbf{.553}$$

12H-50.

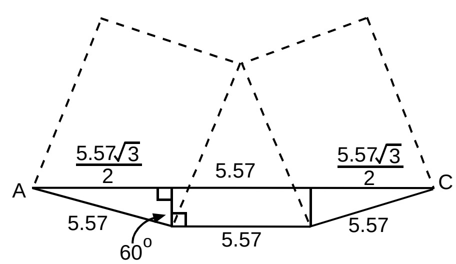
CONGRUENT CUBES AND EQUILATERAL TRIANGULAR PRISM



AB = ?

12H-50 = ________________________

Top view:



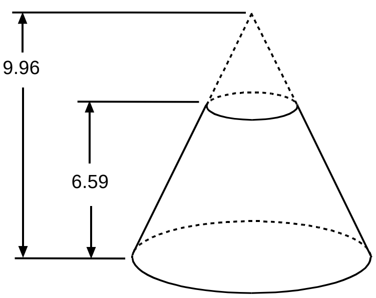
$$AC = 15.217\ldots$$

$$AC^2 + 5.57^2 = AB^2$$

$$AB = \mathbf{16.2}$$

12I-50.

CONES AND FRUSTUM



(Frustum Volume)/(Small Cone Volume) = ?

12I-50 = ________________________

$V_1 = 1/3\ \pi\ H\ [r_1^2 + r_2^2 + r_1 r_2]$

$V_2 = 1/3\ \pi\ r_1^2\ h$

$$\frac{V_1}{V_2} = \frac{H}{h}\,\frac{(r_1^2 + r_2^2 + r_1 r_2)}{r_1^2}$$

$$\frac{r_1}{9.96 - 6.59} = \frac{r_2}{9.96}$$ If $r_1 = 1$, $r_2 = 2.955\ldots$

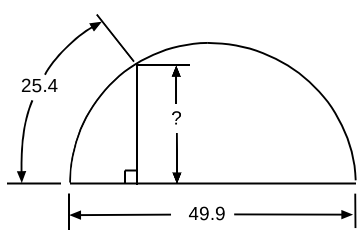
$$\frac{V_1}{V_2} = \frac{6.59}{9.96 - 6.59}\left[\frac{1 + r_2^2 + (1)r_2}{1^2}\right] = \mathbf{24.8}$$

OR

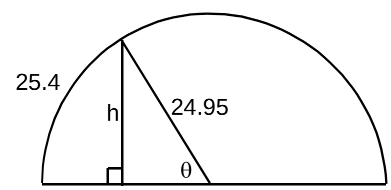
$$\left[\frac{9.96}{9.96 - 6.59}\right]^3 - 1 = \mathbf{24.8}$$

13C-60.

SEMICIRCLE



13C-60 = ________________________



$s = \theta r$

$25.4 = \theta(24.95)$

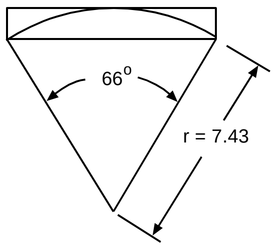
$\theta = 1.018\ldots$ rad

$\sin\theta = h/\ 24.95$

$h = \mathbf{21.2}$

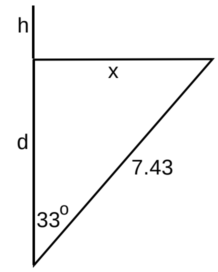
13E-60.

SECTOR AND RECTANGLE



Rectangle Area = ?

13E-60 = ______________________



$\sin 33° = x / 7.43$

$x = 4.046668$

$\cos 33° = d / 7.43$

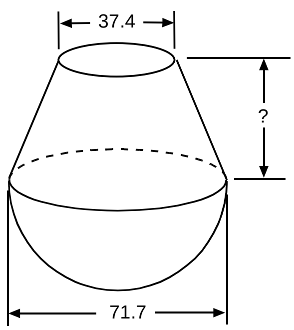
$d = 6.2313\ldots$

$h = 7.43 - d = 1.19867\ldots$

$(2x)h = \mathbf{9.70}$

13F-50.

HEMISPHERE AND FRUSTUM



Volume (Frustum) = Volume (Hemisphere)

13F-50 = ______________________

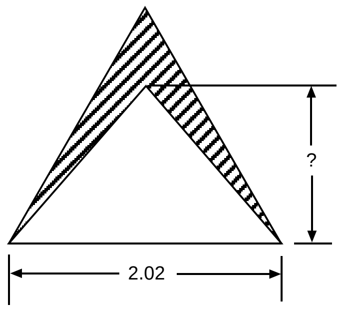
$$\frac{1}{3}\pi\left[\left(\frac{37.4}{2}\right)^2+\left(\frac{71.7}{2}\right)^2+\left(\frac{37.4}{2}\right)\left(\frac{71.7}{2}\right)\right]h$$

$$=\frac{2}{3}\pi\left(\frac{71.7}{2}\right)^3$$

$h = \mathbf{40.0}$

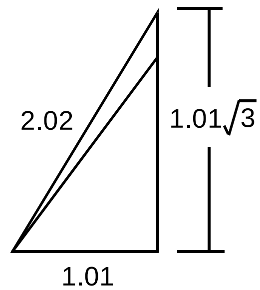
13F-60.

EQUILATERAL TRIANGLE AND TRIANGLES



Hatched Area = 0.464

13F-60 = ________________________

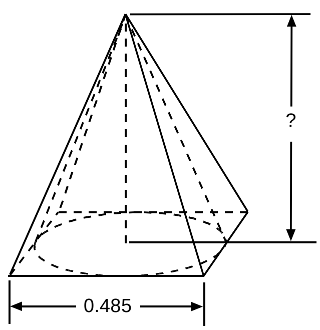


$$\frac{(2.02)^2\sqrt{3}}{4} - 1.01h = .464$$

$h = \mathbf{1.29}$

13G-50.

CONE IN SQUARE PYRAMID



Volume(Pyramid) - Volume(Cone) = 0.00909

13G-50 = ________________________

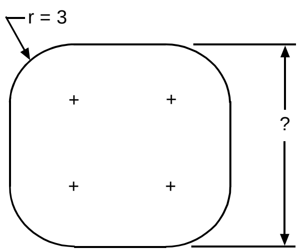
$1/3\ B_1h\ -\ 1/3\ B_2h$

$1/3(.485)^2h\ -\ 1/3\pi(.485/2)^2h\ =\ .00909$

$h\ =\ \mathbf{.540}$

13I-60.

SQUARE WITH ROUNDED CORNERS



Area = 122

13I-60 = ________________________

4(qtr. circles) + 1 sq. + 4(rect.)

$\pi(3)^2 + x^2 + 4(3x) = 122$

$x = 5.4$

$x + 6 =$ **11.4**